

Counting the ways!

Note! ${}^n P_r$ is the second function on the multiplication key, ${}^n C_r$ is the second function on the division key and $x!$ is the key under the ON key.

1 Calculate the value of ${}^8 P_3$, ${}^8 C_5$ and $8!$

2 I am going to burn three of my 12 favorite songs on a CD.

(a) How many different sets of 3 songs are possible from my 12 favorite songs?

(b) In how many ways can I choose 3 songs and put them in order, first, second and third on the CD?

3 A private gym club has 24 members; 15 females and 9 males. Twins Llewellyn and Sandra are two of the female members.

(a) In how many different ways can the letters in Llewellyn be arranged?

(b) A seven person team is to be made from the members. How many teams can be made if:

(i) there is no restriction on the members

(ii) there must be 4 female and 3 male members

(iii) Llewellyn must be on the team

(iv) the team must include one of the twins, another two women and 4 men?

4 Grant has 10 movie DVDs. Three of the DVDS contain romances, five contain mysteries and two contain thrillers. In how many ways can Grant arrange the DVDs if he places

(a) all the mysteries together.

(b) a thriller at each end.

(c) all the romances together, all the mysteries together and both thrillers together?

(d) the mysteries in every second position?

5 Eight people, four men and four women are to be seated at a round table.

(a) In how many different ways can the 8 people be seated?

(b) Calculate the number of ways they can be seated if no two men are sitting next to each other.

6 Six people entered a train carriage with 15 spare seats. In how many different ways can the six people be arranged in the 15 seats?

7 Curried prawns, Mongolian lamb and six other main meals are to be served at a Chinese banquet.

(a) In how many ways can the main meals be served if the curried prawns is served immediately before the Mongolian lamb with no dishes in between?

(b) In how many ways can the main meals be served if the curried prawns is served before the Mongolian lamb?

8 There are 10 digits 0 to 9, which can be used in a PIN. How many 6 digit PINS have the digits in descending order?

Practising unit conversion

1 Complete the missing values.

(a) 1 metre = _____ cm

(b) 1 cm = _____ mm

(c) 1 km = _____ m

(d) 1 gram = _____ mg

(e) 1 kg = _____ g

(f) 1 tonne = _____ kg

(g) 1 Litre = _____ mL

(h) 1 kL = _____ L

(i) 1 hour = _____ min

(j) 1 day = _____ hours

(k) 1 year = _____ weeks

(l) 1 year = _____ days

2 Use your calculator to convert the following length measurements.

(a) 1200 m = _____ km

(b) 55 cm = _____ m

(c) 2.6 km = _____ m

(d) 2500 mm = _____ m

(e) 700 m = _____ km

(f) 600 m = _____ cm

3 Use your calculator to complete each mass statement.

(a) 2.7 t = _____ kg

(b) 300 kg = _____ t

(c) 20 kg = _____ g

(d) 50 000 g = _____ t

(e) 0.04 t = _____ g

(f) 24 kg = _____ t

4 Complete the following time statements.

(a) 13 days = _____ hours

(b) 12 weeks = _____ days

(c) 2 hours = _____ min

(d) 480 h = _____ days

(e) 18 fortnights = _____ weeks

(f) 150 s = _____ min

(g) 12 fortnights = _____ days

(h) 1.75 years = _____ months

5 Use the  button on your calculator to express each time in hours.

(a) 4 h 30 min = _____ h

(b) 6 h 20 min 32 s = _____ h

(c) 2 h 25 min = _____ h

6 Use the Shift then  button to express each decimal time in hours, minutes and seconds.

(a) 4.2 hours = _____ h _____ min

(b) 3.64 h = _____ h _____ min _____ s

7 Express 4.58 hour in hours and minutes, correct to the nearest minute.

8 It takes the planet Saturn 29.46 Earth years to travel once around the sun.

(a) How many Earth hours does it take Saturn to travel once around the sun?

(b) An average day on Saturn is 10.2 Earth hours long. How many Saturn days are there in a Saturn year? (i.e. the number of Saturn days it takes the planet to travel once around the sun.)

Calculating blood alcohol concentrations

Many factors including the amount of alcohol consumed, body weight and the percentage of fluid in the body affect a person's blood alcohol concentration (BAC). One formula for estimating BAC immediately after drinking is:

$$\text{BAC} = \frac{0.0637 \times \text{quantity of alcohol consumed in mL}}{\text{weight of fluid in the body in kg}}$$

The quantity of fluid in a female body changes, but is usually about 49%. Male bodies are constantly 58% fluid.

- 1** Michael's body mass is 75 kg. How many kilograms of fluid are in his body?
- 2** Julia weighs 52 kg. How many kilograms of fluid are in her body?
- 3** A 275-mL bottle of Bacardi Breezer contains 4.8% alcohol. How many milliliters of alcohol are in the bottle?
- 4** Galliano contains 30% alcohol. Max drank three 50-mL drinks of Galliano. How many mL of alcohol did Max drink?
- 5** Natasha's body mass is 68 kg. In one hour she drank three 150-mL glasses of wine that contained 13.5% alcohol.
 - a** How many mL of alcohol did Natasha drink?
 - b** Calculate the weight of fluid in Natasha's body.
 - c** Calculate Natasha's blood alcohol concentration.
- 6** Stephen consumed six 375-mL cans of beer containing 3.8% alcohol. Stephen's body mass is 59 kg.
 - a** How many mL of alcohol did Stephen consume?
 - b** How many kg of fluid are in his body?
 - c** Calculate Stephen's BAC.
- 7** Peter and Elizabeth's body mass is 72 kg. During the same time they both consumed four 275-mL drinks containing 4% alcohol. Calculate their blood alcohol concentration.
- 8** Jessica and Grant both have full driving licenses and they must have a BAC under 0.05 to drive a car. Jessica's body mass is 51 kg and Grant's is 96 kg. During the same time Jessica consumed one 50-mL glass of spirits containing 40% alcohol and Grant drank four 275-mL glasses of 3.9% alcohol beer. Will either of them be able to drive a car legally?
- 9** Use the formula for BAC to explain why the female will always have a higher BAC, if a male and a female with the same body mass consume the same amount of alcohol in the same time.

Investigating number patterns

1 Use your calculator to determine the missing values.

$$11^2 = \underline{\hspace{2cm}}$$

$$111^2 = \underline{\hspace{2cm}}$$

$$1111^2 = \underline{\hspace{2cm}}$$

2 Look at the pattern in question 1.

(a) Predict the value of $11\,111^2$, then check the accuracy of your prediction on your calculator.

(b) A scientific calculator displays the value of $1111\,111^2$ in scientific notation.
Write the value of $1111\,111^2$ without using scientific notation.

(c) Predict the value of $\sqrt{123456787654321}$ then check your prediction on your calculator.

(d) Explain why the value of $1111\,111\,111^2$ is different to the other values in the pattern.

3 (a) Use your calculator to find the following values.

$$101^2 = \underline{\hspace{2cm}}$$

$$102^2 = \underline{\hspace{2cm}}$$

$$103^2 = \underline{\hspace{2cm}}$$

(b) Describe how you could use the pattern to determine the value of 104^2 without using a calculator.

(c) Write down the value of 109^2 without using a calculator, then check your answer on your calculator.

(d) Use the fact that $13^2 = 169$ to determine the value of 113^2 .

4 Look at the following pattern.

$$\left(1\frac{1}{2}\right)^2 = 2\frac{1}{4} \quad \text{Or } 1 \times 2 + \frac{1}{4}$$

$$\left(2\frac{1}{2}\right)^2 = 6\frac{1}{4} \quad \text{Or } 2 \times 3 + \frac{1}{4}$$

$$\left(3\frac{1}{2}\right)^2 = 12\frac{1}{4} \quad \text{Or } 3 \times 4 + \frac{1}{4}$$

(a) Use the pattern to determine the value of $\left(9\frac{1}{2}\right)^2$.

(b) Use the pattern to determine $\sqrt{\left(72\frac{1}{4}\right)}$ and check your answer on your calculator.

5 Challenge! Explain why the patterns on this page work.

Calculator speed and accuracy trial

Test your calculator speed and accuracy with this quick quiz.

Start the clock now!

1 Evaluate each expression correct to 3 decimal places.

(a) $\tan 48^{\circ}23'$

(b) $\sqrt{11.5^2 - 4.6^2}$

(c) $\frac{15.75 + 12.83}{1.2 \times 5.8}$

(d) $(6.32)^{\frac{3}{4}}$

(e) $\sqrt[3]{136}$

(f) $\sqrt[4]{1587}$

2 What is the value of A when $\sin A = 0.875$?

Answer in degrees and minutes correct to the nearest minute.

3 Express $\frac{7}{16}$ as a decimal.

4 Express $5\frac{11}{32}$ as an improper fraction.

5 Write 0.385 as a fraction.

6 Express 471 minutes in hours and minutes.

7 What is the value of π correct to 6 decimal places?

8 Write the value of $2\frac{3}{4} \div 1\frac{5}{8}$ as a mixed numeral.

9 Calculate the mean and standard deviation of the scores in the table.

Write each value correct to two decimal places.

Score	Frequency
11	8
12	15
13	27
14	6

10 Write the value of $(4.9 \times 10^8) \times (1.65 \times 10^{-2})$ in scientific notation correct to 4 significant figures.

Assess your performance!

Calculator wizard All answers correct in less than 7 minutes

Calculator champion All answers correct between 7 and 10 minutes

Remember! Accuracy is more important than speed. Practise the style of any questions you got wrong.

Answers

Counting the ways!

1 ${}^8P_3 = 336$, ${}^8C_3 = 56$, $8! = 40\,320$ 2(a) ${}^{12}C_3 = 220$ (b) ${}^{12}P_3 = 1320$

3(a) $\frac{9!}{4!2!} = 7560$ (b)(i) ${}^{24}C_7 = 345\,104$ (ii) ${}^{15}C_4 \times {}^9C_3 = 114\,660$ (iii) ${}^{23}C_6 = 100\,947$

(iv) ${}^2C_1 \times {}^{13}C_2 \times {}^9C_4 = 19\,656$

4(a) $6! \times 5! = 86\,400$ (b) $2 \times 8! \times 1 = 80\,640$ (c) $3! \times 3! \times 5! \times 2! = 8640$ (d) $10 \times 4! \times 5! = 28\,800$

5 (a) $7! = 5040$ (b) $1 \times 3! \times 4! = 144$ 6 ${}^{15}P_6 = 3\,603\,600$

7(a) $7! = 5040$ (b) $(7 + 6 + 5 + 4 + 3 + 2 + 1) \times 6! = 20\,160$ 8 ${}^{10}C_4 = 210$

Practising unit conversion

1(a) 100 (b) 10 (c) 1000 (d) 1000 (e) 1000 (f) 1000
(g) 1000 (h) 1000 (i) 60 (j) 24 (k) 52 (l) 365 or 366

2(a) 1.2 (b) 0.55 (c) 2600 (d) 2.5 (e) 0.7 (f) 60 000

3(a) 2700 (b) 0.3 (c) 20 000 (d) 0.05 (e) 40 000 (f) 0.024

4(a) 312 (b) 84 (c) 120 (d) 20 (e) 36 (f) 2.5
(g) 168 (h) 21

5(a) 4.5 (b) $6.34\bar{2}$ (c) $2.41\bar{6}$

6(a) 4 h 12 min (b) 3 h 38 min 24 s

7 4 h 35 min 8(a) 258 000 (b) 25 300

Calculating blood alcohol concentrations

1 43.5 2 25.48 3 13.2 4 45

5(a) 60.75 (b) 33.32 kg (c) 0.116 6(a) 85.5 (b) 34.22 (c) 0.159

7 Peter 0.067 Elizabeth 0.0794 8 Jessica 0.051 Grant 0.049. Grant can drive home.

9 Males have more fluid in their body. The denominator of the formula will be larger for males. Dividing by a larger amount decreases the answer.

Investigating number patterns

1 121 12321 1234321

2(a) 123454321 **(b)** 1234567654321 **(c)** 11 111 111

(d) It would be 123456789(10)987654321 which is 1234567900987654321.

3(a) 10201 10404 10609 **(b)** $10(2 \times 4)0(4^2) = 10\ 816$ **(c)** $10(2 \times 9)0(9^2) = 11\ 881$

(d) $10(2 \times 13)0(13^2) = 12600 + 169 = 12\ 769$

4(a) $90\frac{1}{4}$ **(b)** $72\frac{1}{4} = 8 \times 9 + \frac{1}{4}$, $\sqrt{72\frac{1}{4}} = 8\frac{1}{2}$

Calculator speed and accuracy trial

1(a) 1.126 **(b)** 10.540 **(c)** 4.106 **(d)** 3.986 **(e)** 5.143 **(f)** 6.312

2 $61^\circ 3'$ **3** 0.4375 **4** $\frac{171}{32}$ **5** $\frac{77}{200}$ **6** 7 hours 51 min

7 3.141593 **8** $1\frac{9}{13}$ **9** mean = 12.55 standard deviation = 0.86 **10** 8.085×10^6